





4: MA0723

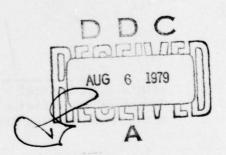
QUASI-STATIC CRACK GROWTH IN METALS AT ELEVATED TEMPERATURE - A REVIEW

Metals Behavior Branch Metals and Ceramics Division

March 1979

TECHNICAL REPORT AFML-TR-78-136

Final Report for Period June 1977 - September 1978



DOC FILE COPY

Approved for public release; distribution unlimited.

AIR FORCE MATERIALS LABORATORY AIR FORCE WRIGHT AERONAUTICAL LABORATORIES AIR FORCE SYSTEMS COMMAND WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

THEODORE NICHOLAS, Project Engineer

NATHAN TUPPER, Chief

Metals Behavior Branch

Metals and Ceramics Division

"If your address has changed, if you wish to be removed from our mailing list, or if the addressee is no longer employed by your organization please notify AFML/LIN _,W-PAFB, OH 45433 to help us maintain a current mailing list".

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

AIR FORCE/56780/19 July 1979 - 410

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE					
1. REPORT NUMBER AFML-TR-78-136 4. TITLE (and Substitle) QUASI-STATIC CRACK GROWTH IN METALS AT ELEVATED TEMPERATURE - A REVIEW 7. AUTHOR(**) L.S. Fu 2. GOVT ACCESSION NO. 3. RECIPIENT'S CATALOG NUMBER 5. TYPE OF REPORT & PERIOD COVER Final Report, June 77—Sep 6. PERFORMING ORG. REPORT NUMBER 8. CONTRACT OR GRANT NUMBER(**)					
AFML-TR-78-136	A CALL OF THE STATE OF THE CASE				
4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERE				
QUASI-STATIC CRACK GROWTH IN METALS AT ELEVATED	Final Report, June 77 - Sep				
TENTORE - A RETTER	6. PERFORMING ORG. REPORT NUMBER				
	8. CONTRACT OR GRANT NUMBER(s)				
Air Force Materials Laboratory/LLN Air Force Systems Command	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS				
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE				
Air Force Materials Laboratory/LLN	March 1979				
Air Force Systems Command	The state of the s				
Wright-Patterson Air Force Base, OH 45433					
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. PECURITY CLASS. (of this report)				
(16) 23×7/ (12) P1/					
have been a second	15a. DECLASSIFICATION DOWNGRADING				
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different fro	m Report)				
18. SUPPLEMENTARY NOTES The author is now Associate Professor, Department of University, Columbus, Ohio.	Eng. Mech., The Ohio State				
19. KEY WORDS (Continue on reverse side if necessary and identify by block number,					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep Crack Growth					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep Crack Growth					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep Crack Growth Elevated Temperature					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep Crack Growth Elevated Temperature 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)					
19. KEY WORDS (Continue on reverse side if necessary and identify by block number, Creep Crack Growth Elevated Temperature 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The literature on fatigue and sustained load crack of temperature is examined within the framework of fractional crack growth (FCG) rates are found to be frequency of from transgranular to intergranular failure modes is the time-dependent effect on FCG rates comes from oxportack growth. In extrapolating the low-cycle fatigue	rowth in metals at elevated ture mechanics. Fatigue ependent. A transition noted as frequency decreas idation damage and creep to laboratory test data to				
Dreep Crack Growth Elevated Temperature 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The literature on fatigue and sustained load crack (temperature is examined within the framework of fractional crack growth (FCG) rates are found to be frequency of from transgranular to intergranular failure modes is the time-dependent effect on FCG rates comes from on	rowth in metals at elevated ture mechanics. Fatigue ependent. A transition noted as frequency decreas idation damage and creep to laboratory test data to				

DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Deta Entered)

0/2 320

prediction of initiation and growth of cracks are critically reviewed. Rest are not conclusive as to which fracture parameter should be used for predict Guidelines are given for further studies in this area.	modify the predicted behavior. Question at stress in fracture toughness. Keep Empiric	tensity levels well below the al results and theoretical m	e materials odels for
CEASURE STORE AND ARTHUR STORE CONTRACTOR AND ARTHUR STORE	are not conclusive as to which fr	acture parameter should be u	reviewed. Result sed for prediction
CEASURE STATE AND ART STATE AR		A	
Torce Present College			
Contraction of the second of t			
Force System Landson Force System Landson The Contract Contract Contract The Contract Contract Contract Contract The Contract Contract Contract Contract Contract Contract The Contract C			
to Tiles of any and the second			
The production of the producti			
		Bind antibulisty to preselve to	
COLOURS CONTROL OF THE CONTROL OF TH			
Approximately and the second s			

FOREWORD

This technical report was prepared by the Metals Behavior Branch, Metals and Ceramics Division, Air Force Materials Laboratory. The work was performed under in-house Project No. 2307Pl while the author was a Senior Research Associate with the National Research Council. The report covers work conducted from June 1977 to 1978 and was submitted in September 1978.

The author is grateful to Dr. T. Nicholas, Air Force Materials Laboratory for consultation during his six-month tenure.

NTIS (
Jnanno		
Justif	leation	
Bv		
*	bution/	
	ability Ond	0.5
1	Availand/o	
Dist	special	1
A	special	
1		
H		

AFML-TR-78-136

TABLE OF CONTENTS

SECTI	ON		PAG
I	INT	RODUCTION	1
II	FAT	IGUE-CREEP INTERACTION	2
	1.	Time-Independent and Time-Dependent Effects on FCG Rates	2
	2.	Modified Linear Superposition Model	4
	3.	Summary	5
111		ROSCOPIC DESCRIPTION AND EMPIRICAL RESULTS CREEP CRACK GROWTH	7
	1.	The Uniaxial Creep Behavior	7
	2.	Microscopic Description of Creep Crack Growth	8
	3.	Experimental Work and General Behavior	12
	4.	Data Correlation	18
	5.	Summary	19
VI	MAC	ROSCOPIC DESCRIPTION OF CREEP CRACK GROWTH	26
	1.	Critical Strain Model	26
	2.	Model of Critical Plastic Zone Size	29
	3.	Continuous Rupture Model	32
	4.	Model of C*-Integral	32
	5.	Critical COD Model	35
	6.	Summary	38
٧	CON	ICLUSIONS	39
	REF	FRENCES	40

LIST OF ILLUSTRATIONS

IGUE	RE	PAGE
1	Schematic Representation of Log (A (ν ,T) Versus Log ($1/\nu$) at Constant Temperature Illustrating Various Time-Dependent Contributions to the Proportionality Constant A (ν ,T)	6
2	Creep Strain Versus Time	7
3	Creep Crack Growth Versus Time	9
4	Cross Section of Interrupted Test Showing Grain Boundary Microcracks	10
5	Temperature to Produce Rupture in High-Temperature Alloys After 100 and 1000 Hours at a Stress of 140 MPa	11
6	One Hundred Hour Rupture Strength in Several Cast Superalloys as a Function of Temperature	12
7	Displacement Data From Tests (T-L Crack Orientation) of 2219-T851 Plate (3-In-Thick) at 300°F	14
8	Crack Length Versus Time for 2219-T851 Plate (3-In-Thick) at 300°F	14
9	K Versus Time for 2219-T851 Plate (3-In-Thick) at 300°F	15
10	Fracture Surface of a Specimen Alternately Fatigue Cracked and Creep Loaded for Different Intervals of Time	16
11	Time to Failure Versus Initial Stress Intensity for Samples in Longitudinal Direction Tested at 704°C	17
12	Time to Failure Versus Initial Stress Intensity for Inconel Alloy 718 at Various Temperatures	17
13	Schematic Illustration of Crack Growth Data	18
14	Normalized Stress and Strain Around a Groove Under Plane-Strain Tension (Mode I)	20
15	Effect of Temperature on Creep Crack Growth Behavior in Alloy 718	24
16	A Comparison of the Ratio Stress Intensity Factor/ Sketal Point Stress for Various Specimen Geometries	25

LIST OF ILLUSTRATIONS (CONTINUED)

FIGL	JRE	PAGE
17	Comparison of the Crack Growth Rate \dot{a}_2 Obtained by Predictions of the Model, With Crack Growth Rate \dot{a}_1	
	Predicted by the Empirical Correlation With the Elastic Stress Intensity Factor	28
18	The Crack Growth Rate Against Stress Intensity Factor Curve	31
19	Schematic Showing Six Steps Involved in C* Data Reduction	34
20	Variation of m, dm/d (a/w), and (1/m), $(dm/d(a/w))$ with (a/w)	36

SECTION I

INTRODUCTION

Crack growth behavior in metals at elevated temperature has recently received much attention. This is due to the fact that a severe degradation in fatigue resistance results under conditions of low frequencies or long hold times for certain materials. It has been observed, for example, that uniaxially loaded test specimens of AISI 304 stainless steel tested at 650°C with a controlled cyclic strain range of 0.5% and hold times of one hour failed in less than 600 cycles, whereas without the hold times, but with the same strain range, the life time was some 15,000 cycles (Reference 1). Similar effects have been reported under conditions where time effects were introduced by stain rate (Reference 2) or frequency of cycling (References 3,4).

Since elevated temperature low-cycle fatigue is a major design limitation in the application of metals to elevated temperature service, to extrapolate the low-cycle fatigue laboratory test data to service conditions of interest it is essential that the influence of hold times and cycling frequency be taken into account. This is an extremely important aspect of the elevated temperature fatigue behavior, since the interaction with creep, oxidation and structural degradation at elevated temperature can drastically modify the predicted behavior.

In this report, the fatigue-creep interaction is first reviewed and summarized. The microscopic approach (micromechanism) to creep crack growth (CCG) is then briefly reviewed and discussed. Finally, the macroscopic (phenomenological) approach to CCG (crack growth under sustained load) is reviewed and discussed in detail. Crack growth studies that employ a viscoelastic analysis apply mostly to polymers and therefore are not included in the present survey.

SECTION II

FATIGUE-CREEP INTERACTION

TIME-INDEPENDENT AND TIME-DEPENDENT EFFECTS ON FATIGUE CRACK GROWTH RATES

The most limiting properties in the preliminary design phase for metals applied at elevated temperature are ultimate and stress rupture strengths. Subsequent detailed design of components involves life analysis, and materials acceptance for this purpose involves fatigue, creep and fracture mechanics data. It is of importance to establish methods for predicting failure under combined creep-fatigue conditions from short-term laboratory test data.

Ignoring overload effects (retardation), the three important parameters that influence fatique crack growth (FCG) rates are: frequency (v), cycle or stress ratio (R) and temperature (T). It has long been established that FCG rates may be correlated with cyclic range in stress intensity factor by the power law of Paris (Reference 5). Recent studies on low-cycle FCG rates at elevated temperature (Reference 6) demonstrated the applicability of the power law form of the stress intensity factor, K, as a governing parameter in crack growth rate using the power law. Wallace, et al. (Reference 7) have found success in studying low-cycle FCG rate in a nickel-base superalloy IN-100 by using a hyperbolic sine (SINH K) model instead of a power law model. This implies that at high frequency, say 10 Hz or more, the FCG rate at elevated temperature is cycle dominant. Elevated temperature (1100°F) FCG results in air at frequencies in excess of 10 Hz. Pelloux and Grant (Reference 8) on HS-188 also indicate a time-independent, cycle dominant FCG characterized by the purely transgranular nature of the crack propagation.

Time-dependent effects on the FCG rate at elevated temperature appear only at lower frequencies. Furthermore, there is an overwhelming body of evidence developed in recent years to show that the time-dependent

phenomena such as creep and oxidation that occur at decreasing cycling frequency at elevated temperature are responsible for the severe degradation in fatigue resistance under those conditions.

In materials with poor oxidation resistance, additional crack growth can occur as a consequence of the oxidation damage at the crack tip. The FCG rates would be strongly frequency dependent. For example, Solomon and Coffin (Reference 3) observed such a dependence on frequency in A286 for FCG at elevated temperature (1100°F) while conducting tests in air. The effect is drastically decreased if tests are conducted in vacuum. James (Reference 9) investigated the effect of frequency upon the FCG of 304 stainless steel at 1000°F. Later, James (Reference 10) discussed two different thermally-activated time dependent processes and concluded that oxidation damage at elevated temperature is responsible for the increase in FCG at increasing temperature. The mechanism with which the oxidation damage at the crack tip can accelerate the intergranular FCG rates has been discussed by Gell and Leverant (Reference 11). The rate controlling mechanism is either the surface diffusion of oxygen to the crack tip or the diffusion of oxygen (either through the lattice or along the grain boundaries) ahead of the crack tip. In either case an activation energy $\mathbf{Q}_{\mathbf{n}\mathbf{x}}$ can be determined from an Arrhenius plot that characterizes the process. It appears that there is an upper and lower bound on frequency for oxidation damage, i.e., oxidation damage becomes insignificant at both high and low ends of the frequency range. However, Solomon (Reference 4) showed that this transition region of oxidation influence can extend over more than four orders of magnitude in cycling frequency for materials that are poor in oxidation resistance such as A286 and titanium.

At very long cycle periods or very low frequencies, the FCG rates become purely time-dependent (Reference 12). The crack propagation is characterized by its intergranular nature, and the rate-controlling mechanism is most likely one that involves the growth and coalescence of grain boundary voids by vacancy condensation (Reference 13).

Sufficient experimental evidence is now available to suggest that the creep crack growth (CCG) process is characterized by a single activation energy \mathbf{Q}_{Cp} (Reference 14), obtained from the slop of an Arrhenius plot. This activation energy is about 0.7 - 0.8 of that of the steady-state creep, for example, in superalloys (Reference 14). This activation energy \mathbf{Q}_{Cp} should preferably be determined from CCG experiments (Reference 15).

There has been some concern over the application of linear elastic fracture mechanics to CCG. Since significant time-dependent plastic deformation occurs during creep, several authors (References 16, 17, 18) proposed the use of a potential energy release rate, J or C* integral for the characterization of crack tip behavior, while others proposed the use of crack tip opening displacement (CTOD (References 19, 20, 21). Other approaches include the net-section stress, $\sigma_{\rm net}$ (References 22, 23, 24) and the crack tip opening angle (CTOA) (Reference 25). However, if the metal behaves in a macroscopically brittle fashion under creep conditions, the stress intensity factor K computed by linear elastic theory can be used to characterize the crack tip configuration. For example, Floreen (Reference 14) found for a variety of nickel-base superalloys that the CCG rate was proportional to an exponential power of the stress intensity factor K. Siverns and Price (Reference 26) showed a similar CCG rate dependence on K for a quenched 2-1/2-Cr-1 Mo steel tested in tension at 565°C. A review of experimental and theoretical work on CCG is given in the next section where the question of crack tip characterization under creep conditions will be discussed.

MODIFIED LINEAR SUPERPOSITION MODEL

It is clear from the previous discussions that the fatigue-creep interaction is time-dependent with cycle dominance at very high frequency, in excess of 10 Hz, with creep dominance at very low frequency, less than 0.1 Hz, and with a transition region where oxidation damage

contribution may or may not appear depending upon the material resistance to oxidation. A model for FCG at elevated temperature can be proposed as:

$$da/dN = C_{0}(0)f^{m}(K,R,n)$$

$$+C_{1}(0)f^{m}(K,R,n)(1/v)^{k} \exp(-\frac{Q_{ox}}{RT})$$

$$+C_{2}(0)f^{m}(F,R,n)(1/v) \exp(-\frac{Q_{cp}}{RT})$$
(1)

where \mathbf{C}_0 , \mathbf{C}_1 , \mathbf{C}_2 , m, and n are empirical material constants. The functional form of f may be the conventional power law such as:

$$f^{m}(K,R,n) = \Delta K^{m}(1-R)^{m+2n}$$

or it may be a hyperbolic sine function as suggested by Annis, et al. (Reference 27). The parameter F is the proper crack tip characterization factor under the particular creep conditions for a given material. Such a linear superposition model was first proposed by Carden (Reference 12) and modified by Doner (Reference 15). For the simplicity of illustration, let F = K. Equation 1 is then reduced to:

$$\frac{da}{dN} = A(v,T)f^{m}(K,R,n)$$
 (2)

A schematic representation of A(ν ,t) as a function of (1/ ν) is given in Figure 1.

3. SUMMARY

In summary, the fatigue-creep interaction exclusive of retardation effect is as follows:

- a. For given T and ΔK , the FCG rate (da/dN) increases with increasing temperature (References 8, 28, 7).
- b. For given T and ΔK , the FCG rate (da/dN) increases with decreasing frequency (References 8, 9) and increasing hold-times (References 29, 30).

At each temperature, there appears to be a critical frequency $(\nu_{transition})$ above which da/dN is more or less frequency independent. The $\nu_{transition}$ increases with increasing temperature and decreasing ΔK (Reference 8).

- c. FCG rate under creep may be written as da/dN = (da/dt)(1/v) for very low frequencies (Reference 8) and/or very long hold-times (References 31, 8), the crack growth rates under creep conditions appear as the lower limit of the FCG rates.
- d. At a given temperature and frequency, da/dN increases with increasing stress ratio R. For low-cycle fatigue testing ΔK may be used for correlation (References 28, 7, 14).
- e. FCG is associated with a transgranular-intergranular fracture mode transition. The fracture mode is entirely transgranular above $v_{\text{transition}} \text{ and predominantly intergranular below } v_{\text{transition}} \text{ (Reference 8)}.$

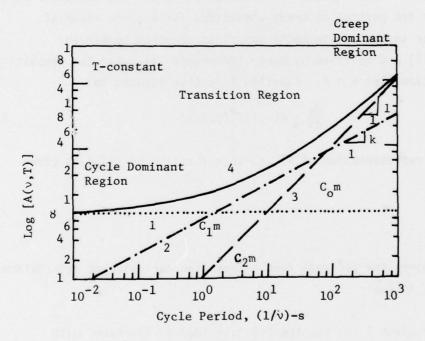


Figure 1. Schematic Representation of Log |A(v,T)| Versus Log (1/v) at Constant Temperature Illustrating Various Time-Dependent Contributions to the Proportionality Constant A(v,T) (Reference 15)

SECTION III

MICROSCOPIC DESCRIPTION AND EMPIRICAL RESULTS OF CREEP CRACK GROWTH

1. THE UNIAXIAL CREEP BEHAVIOR

At elevated temperature, metals often exhibit nonlinear, time-dependent deformation. Under uniaxial tensile loading, the strain in a smooth bar increases with time until failure finally occurs. Based on the similar response of many materials, researchers have subdivided the creep curve into three regions, as shown in Figure 2. After the initial instantaneous strain ε_0 , materials often undergo a period of transient

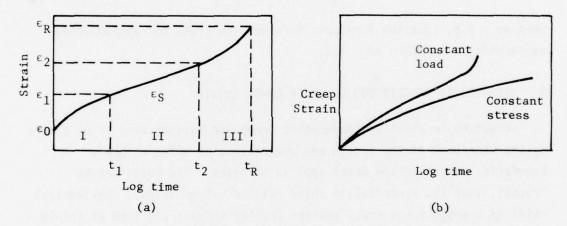


Figure 2. Creep Strain Versus Time

response where the strain rate, de/dt, decreases with time to a minimum steady-state value that persists for a substantial portion of the material's life. These two regions are referred to as transient (primary) stage and steady-state stage, respectively. Final failure with a rupture life t_R occurs soon after the creep rate begins to increase during the third (tertiary) stage of creep. Several empirical relationships between creep strain or creep strain rate and stress have been commonly employed, two of which are:

$$\dot{\varepsilon} = F \sigma^{n}$$
 (3)

and

$$\varepsilon = H\sigma^{p}t^{x}$$
 (4)

where $\dot{\epsilon}$ and ϵ are strain rate and strain, respectively; F, H, n and p are temperature-dependent material constants. The typical values of n and p range from about 1.5 to 10, depending on the stress regime examined, material, and testing conditions. It is now generally recognized that the steady-state creep rate $\dot{\epsilon}_{\rm S}$ varies directly with σ at low stresses and at temperatures near the melting point. At intermediate to high stresses and at temperatures above 0.5 T_m, $\dot{\epsilon}_{\rm S} \propto \sigma$ (Reference 5). A more general empirical relationship has been proposed (Reference 32):

$$\dot{\varepsilon}_{\rm S} = F({\rm Sinh} \ \alpha\sigma)^{\rm n}$$
 (5)

When $\alpha\sigma < 0.8$, Equation 5 reduces to power law creep but approximates exponential creep when $\alpha\sigma > 1.2$.

2. MICROSCOPIC DESCRIPTION OF CREEP CRACK GROWTH

Fracture in elevated temperature creep and fatigue very often occurs by the mechanism of the growth and coalescence of voids at grain boundaries as they glide under applied stresses. The voids may be "round", that is, spherical in shape (called r-type voids). Cylindrical voids of a wedge-shape cross section (called w-type) can form at triple points where three grain boundaries join together along a common line of intersection (References 33, 34).

It has been proposed by several authors that under creep conditions the diffusion of vacancies towards a crack or a void and their condensation there would contribute to crack growth. The migration of vacancies to crack tips and the resulting growth of the cracks have been studied by diffusional and ductile processes by Heald and Williams (Reference 35), Stevens and Dutton (Reference 36), Weertman (Reference 13), Stevens, et al. (Reference 37), and Pals, et al. (Reference 38). The growth of grain-boundary voids by vacancy-condensation has been studied by numerous authors, such as Hull and Rimmer (Reference 39), Speight and Harris

(Reference 40), Dobes and Cadek (Reference 41), Dobes (Reference 42), Weertman (Reference 13), Raj and Ashby (Reference 43), Raj (Reference 44), Speight and Beere (Reference 45), Needham and Greenwood (Reference 46), Needham, et al. (Reference 47), Kelly (Reference 48), and Hancock (Reference 49). A very recent review on analytical treatments of creep crack growth due to vacancy diffusion and condensation is given by Van Leeuwen (Reference 50). On account of stress-modified diffusion equations, he concludes that there will be a transient redistribution of vacancies leading to a steady-state concentration distribution with many vacancies in compressed regions and few in expanded regions. This steady-state vacancy flax is shown to be independent of crack tip stress field but dependent on the strain energy release rate (G). With this vacancy flux an expression is derived for creep crack growth rate due to vacancy diffusion and condensation, which accounts for initial (Stage I) and final (Stage III) behavior of creep crack growth (Figure 3).

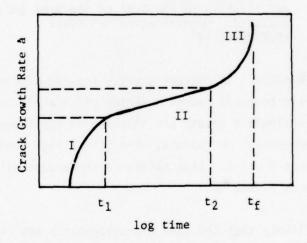


Figure 3. Creep Crack Growth Versus Time

Floreen (Reference 14) showed that quasi-static crack growth in a variety of wrought nickel-base alloys is intergranular in character. A number of test specimens were unloaded short of failure and cross-sectioned vertically to metallographically examine the crack paths. An example of such a section is given in Figure 4. The examinations revealed that one or more microcracks were present on grain boundaries

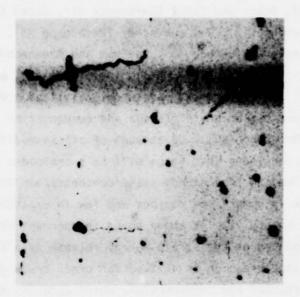


Figure 4. Cross Section of Interrupted Test Showing Grain Boundary Microcracks. Main Crack Is Horizontal and Slightly to the Left of the Area Shown. Unetched. Magnification 320 Times. (Reference 14)

ahead of the main crack. In coarser grains, instead of a distinct microcrack, a grain boundary showed a series of separate holes, suggesting that cavity nucleation and growth was involved in the formation of the microcracks. Information on rupture, strength of high temperature alloys is given in Figures 5 and 6. Discussions on these superalloys can be found in References 51 and 52.

It is very likely that the same micromechanisms are responsible, at least partially, for creep crack growth in other materials through localized deformation. If this is the case, they must operate in limited regions very near to the crack tip and under fairly complex stress conditions. A complete theoretical description of the mechanisms is substantially more complicated than what has been described by the previous authors. Interactions between the main crack and the voids such as that described by Rice and Tracey (Reference 53), Heald, et al., (Reference 54), and Rice and Chuang (Reference 55) should also be studied. Only limited advances have been made so far in this area.

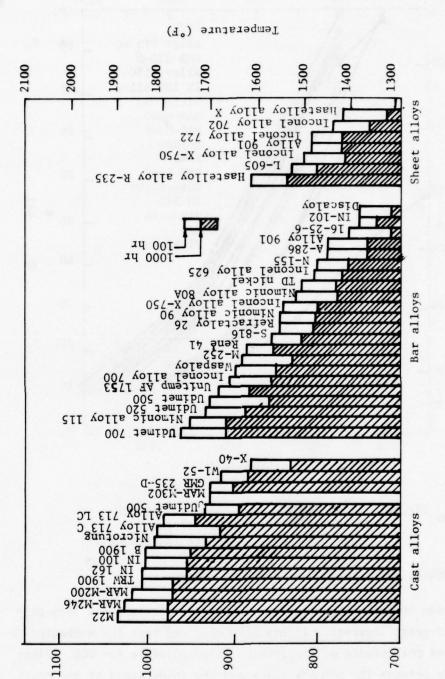


Figure 5. Temperature to Produce Rupture in High-Temperature Alloys After 100 and 1000 Hours at a Stress of 140 MPa (Reference 52)

Temperature (°C)

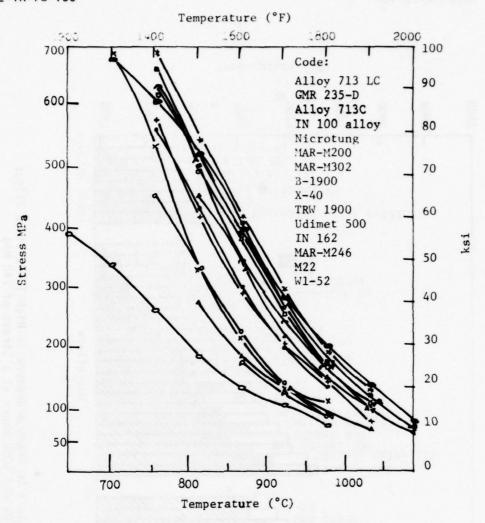


Figure 6. One Hundred Hour Rupture Strength in Several Cast Superalloys as a Function of Temperature (Reference 52)

3. EXPERIMENTAL WORK AND GENERAL BEHAVIOR

The creep behavior of metals at elevated temperature has long been a subject of great interest. It has been observed that the mechanism of growth and coalescence of microcracks is responsible for CCG. Since the distance between the primary crack and the voids ahead of the crack tip is typically of the order of the crack opening displacement, the damage during CGG can be considered to be localized. Several authors

have thus taken a macroscopic approach toward the problem of CCG. Fracture mechanics concepts are used in conjunction with macroscopic creep properties of the material to seek macroscopic criteria governing crack growth behavior.

In this section empirical results that describe crack growth under creep conditions are reviewed. The primary concern here is not only determining the crack growth law in terms of macroscopic parameters but also delineating the proper use of the law for given material and service conditions.

When a fracture mechanics specimen is pre-cracked in fatigue and subjected to a sustained load at constant elevated temperature, quasistatic crack growth and final fracture will occur. As time elapses, the crack opening displacement (COD), the crack length (a), and the apparent stress intensity factor (K) will increase. Typical examples of this behavior are shown in Figures 7-9 (References 14, 26, 56, 57).

It appears that a definite incubation period exists during which no crack occurs. A quasi-static crack growth period follows and then fast fracture occurs. At all stages of crack growth, inclusive of the fast fracture region, the crack front bows out in thumb-nail style and large-scale plasticity occurs (Figure 13). A typical fracture surface of a specimen alternately fatigue cracked and creep loaded is shown in Figure 10 (Reference 82).

While no crack growth is observed at room temperature for a K-level below $K_{\rm IC}$, crack growth is observed at elevated temperatures at stress intensities well below $K_{\rm IC}$ (Figure 8, Reference 56). However, a threshold value of K may occur below which no crack growth takes place within 1000 hours (Figure 11, Reference 14). This threshold value of K may vary slightly depending upon temperature (Figure 12, Reference 14).

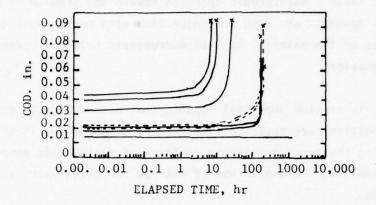


Figure 7. Displacement Data from Tests (T-L Crack Orientation) of 2219-T851 Plate (3-In-Thick) at 300°F (Reference 56)

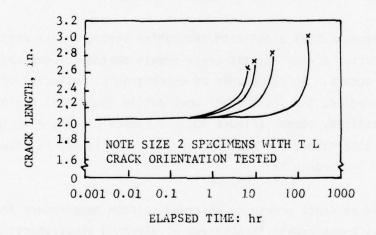


Figure 8. Crack Length Versus Time for 2219-T851 Plate (3-In-Thick) at 300°F (Reference 56)

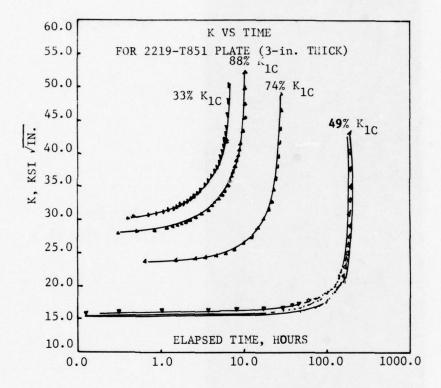


Figure 9. K Versus Time for 2219-T851 Plate (3-In-Thick) at 300°F (Reference 56)

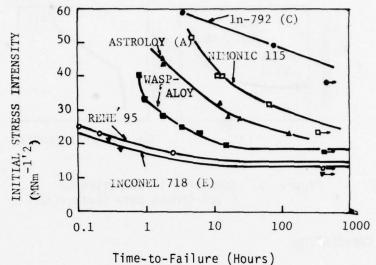


FAST FRACTURE

CREEP CRACK (732°C, 1350°F)

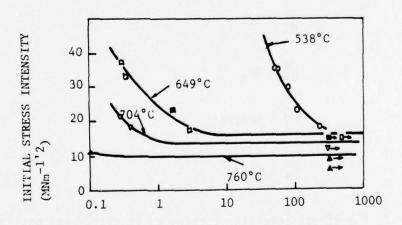
FATIGUE CRACK

Figure 10. Fracture Surface of a Specimen Alternately Fatigue Cracked and Creep Loaded for Different Intervals of Time (Reference 57)



Time co-fatture (nours)

Figure 11. Time-to-Failure Vs Initial Stress Intensity for Samples in Longitudinal Direction Tested at 704°C (Reference 14)



Time-to-Failure (Hours)

Figure 12. Time-to-Failure Vs Initial Stress Intensity for INCONEL Alloy 718 at Various Tempera-tures (Reference 14)

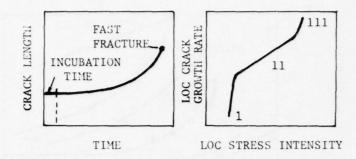


Figure 13. Schematic Illustration of Crack Growth Data (Reference 14)

4. DATA CORRELATION

Although it has been commonly observed that creep crack propagation possesses three distinct Stages, (I, II, and III, i.e. incubation, stable growth and fast fracture (Figure 13)), there has been no general agreement, on the characterization of CCG rate. There are available empirical data supporting any one of the following predictions of CCG rate:

$$\frac{da}{dt} = AK^{\alpha} \tag{6}$$

$$\frac{da}{dt} = B\sigma_n^{\beta} \tag{7}$$

$$\frac{da}{dt} = C(dy/dt)^{\gamma}$$
 (8)

$$\frac{da}{dt} = D(C^*)^{\zeta}$$
 (9)

where K is the stress intensity factor, σ_n is either net section stress, reference stress or skeletal stress, dy/dt is load-line crack opening displacement rate, and C* is a line integral related to the rate of change of potential energy release per unit crack advancement, Haigh (Reference 30) briefly reviewed previous work and gave a table to summarize test results. Van Leeuwen (Reference 50) presented a summary including some references on work using net section stress and the C*-integral.

According to Nicholson (Reference 23) the exponent β in Equation 7 and n in Equation 3 should be equal for the same material at the same temperature while Taira and Ohtani (Reference 58) disagreed. Based upon Townsend's work (Reference 59), Williams and Price (Reference 60) indicated that metals at elevated temperature can be classified as creep brittle (intergranular) if p (or n) is less than 5, and creep ductile (transgranular) if p (or n is greater than 5. The influence of n on stress distribution is given in Figure 14 (Reference 61). In a form similar to that given by Haigh (Reference 30), Table 1 presents such a summary of various materials and geometries and the best correlation with the various equations postulated for CCG.

SUMMARY

In conclusion CCG rates in metals at elevated temperatures are characterized as follows:

- a. CCG rates in Stages I and II increase significantly with an increase in temperature (References 14, 57) (Figure 15).
- b. CCG rates are sensitive to heat treatments and grain size (Reference 14).
- c. The stress exponent p (or n) in the uniaxial creep law plays an important role in determining the characterization of the crack tip behavior. For p (or n) \leq 5 the stress intensity factor approach may be used, and for p (or n) \geq 7 the nominal stress approach may be used (References 14, 22, 60)
- d. Critical test conditions should consist of at least two geometries of different K/σ net ratio (Figure 16) (References 16, 60).
- e. The exponent α in Equation 6 decreases as K increases. This suggests that other functional forms may be used in place of the power law.

Figure 14. Normalized Stress and Strain Around a Groove Under Plane-Strain Tension (Mode I). For n = 0, Solid and Dashed Lines Denote Double and Single Grooves, Respectively

1.0

0.5

0

εrθ

π/2 θ

1.0

0.5

0

 $\sigma_{r\theta}$

π/2 θ

TABLE 1 SUMMARY OF STATIC LOAD TESTS

No.	Reference	Material and Temperature	(00)	Apparent K(MN m ^{-3/2})	s	Geometry*	Best Correlation
-	Popp and Coles ³¹ (1969)	Inconel 718	(238)	100 - 160	7	doo	(9)
2	Siverns and Price ²⁶ (1970, 1973)	2-1/4%CR-1%Mo Quenched Steel	(295)	10 - 90	5.5	SEN (T)	(9)
ю	Harrison and 24 Sandor (1971) ²⁴	1%Cr-Mo-V Wrought Steel	(238)	25 - 60	4.5	CCP	(7)
4	James (1972) ²⁸	Type 316, Cold Worked, Steel	(538)	25 - 40	>20	doo	(9)
2	James (1972) ²⁹	Type 316, Cold Worked, Steel	(538)	25 - 40	>20	CON.K.	(9)
9	Robson (1972) ⁷⁸	0.2%C Cast Steel (NC)	(450)	40 - 50	20	CON.K.	(9)
7	Robson (1972) ⁷⁸	0.2%C Cast Steel (NC)	(450)	40 - 50	20	SEN (T)	(9)
80	Robson (1972) ⁷⁸	0.2% Wrought Steel (C)	(450)	30 - 40	14	CON.K.	(9)
6	Thornton (1972) ⁷⁹	1%Cr-Mo-V Wrought Street (1) .	(292)	30 - 70	80	SEN (C)	(9)
10	Thornton (1972) ⁷⁹	1/2%Cr-Mo-V Cast Steel (2)	(292)	40 - 50	16	SEN (B)	(9)
=	Thornton (1972) ⁷⁹	1/2%Cr-Mo-V Wrought Steel (3) (Coarse Grain)	(295)	17 - 40	9	SEN (B)	(9)
12	Thornton (1972) ⁷⁹	1/2%Cr-Mo-V Wrought Steel (4)	(292)	35 - 50	9	SEN (B)	(9)
13	Neate and Siverns (1973)	2-1/4%Cr-1%Mo Quenched Steel	(293)	10 - 90	5.2	SEN (T)	(9)
14	Neate and Siverns (1973)	1/2Cr-Mo-V Quenched Steel	(999)	12 - 70	3.4	SEN (T) 2 widths	(9)

TABLE 1 (CONTINUED)

No.	Reference	Material and Temperature	(0°)	Apparent K(MN m ^{-3/2})	s	Geometry*	Best Correlation
15	Kenyon et al. ⁶² (1973)	RR58 Aluminum Alloy	(100-200)	17 - 23	30	Constant K	(6)
91	Landes and Wei ⁸⁰ (1973)	4340 Steel	(20-140)	30 - 200		CCP	(9)
11	Ellison and ⁸¹ Walton	1 Cr-Mo-V	(24-565)	1		SEN (T) (B)	(9)
18	Floreen (1975) ¹⁴	Ni-Alloy	(500-750)	1		ct	(9)
19	Haigh (1975) ¹⁹	l Cr-Mo-V Cast Steel	(220)	1		MOL	(8)
20	Nicholson and 22 Formby (1975)	316 Stainless Steel	ŀ	8 - 80		SEN	(7)
12	Kaufman, Bogardus, Mauney and Malcolm (1976)56	2219-T851 Aluminum	(100-149)	11		LJ	(9)
22	Nicholson (1976)23	316SS	(000-820)	I	12.5	DEN	(7)
23	Landes and Begley (1976)	Discalloy	(647)	1		CT,CCP	(6)
24	Shahinian and Sadananda (1976)	Alloy 718	(24-760)	15 - 90		CT,SEN	(6)
52	Nikbin, Webster and Turner (1976) ¹⁷	1/2 Cr-1/Mo-1/4V	(292)	15 - 70		CON.K.	(6)
56	Sadananda Shahinian (1977) ⁵⁷	Alloy 718	(538,649, 760)	20 - 90 ksi - in	1.5	CT	(6)
27	Harper, Ellison (1977) ¹⁸	1Cr-Mo-V Steel	(- 095)	i		CT,SEN	(6)

TABLE 1 (CONTINUED)

Bending	Constant K specimen	Center cracked panel	Compact tension specimen	Double cantilever beam	Double edge notch	Single edge notch	Tension	Wedge opening load
1	1	1	-	1	1	-	-	;
*(B)	Con.K.	doo	5	DCB	DEN	SEN	(E)	MOI

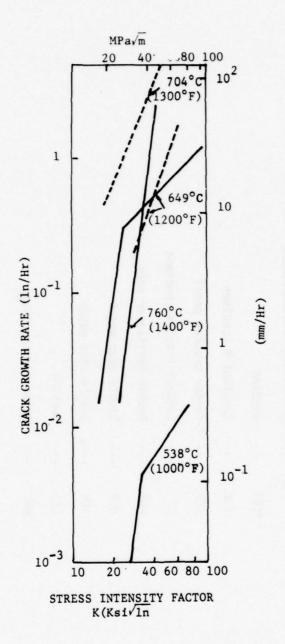


Figure 15. Effect of Temperature on Creep Crack Growth Behavior in Alloy 718 (Reference 57)

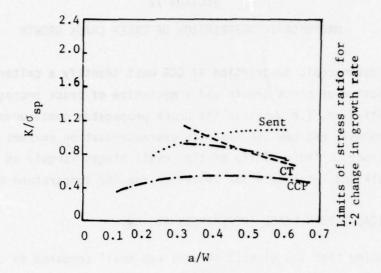


Figure 16. A Comparison of the Ratio $\frac{\text{Stress Intensity Factor}}{\text{Skeletal Point Stress}}$ for Various Specimen Geometries (Reference 60)

- f. Since da/dt may not always be directly proportional to dy/dt, the COD rate criterion may have only limited application (References 19, 21, 61).
- g. CCG results from a combination of two competing processes, i.e. growth and coalescence of defects, which contributes to crack advancement and creep deformation process that causes retardation of crack growth and even its arrest (Figure 15, Reference 57).
- h. Incubation time is reduced with increased initial crack length and by soaking the specimen at the test temperature before loading, indicating both geometry and aging sensitivity (Reference 62).

SECTION IV

MACROSCOPIC DESCRIPTION OF CREEP CRACK GROWTH

Any macroscopic description of CCG must identify a criterion for the initiation of crack growth and a mechanism of crack propagation after initiation, i.e. whether the crack propagates continuously or discontinuously and how the near-tip characterization changes as the crack advances. The validity of the result hinges largely on the modeling of the bulk and local material behaviors for the temperature of concern.

CRITICAL STRAIN MODEL (REFERENCES 63, 64)

Assuming that the elastic strains are small compared to creep strains at steady-state creep, Hoff's (Reference 65) mathematical analogy showed that the creep rates are numerically equal to the elastic strains calculated for the purely elastic case, i.e.

$$\frac{\dot{\varepsilon}_{C}}{\dot{\varepsilon}_{O}} = \frac{\varepsilon_{e}}{\varepsilon_{O}} \left(= \frac{\sigma_{e}}{\sigma_{o}} = \frac{K}{\sigma_{o}(2\pi x)^{1/2}} \right)$$
 (10)

where subscripts C, e and o refer to creep, elastic, and reference states, respectively. Barnby used the above relation and computed the creep stress near the crack tip as

$$\frac{\sigma_{C}}{\sigma_{O}} = \{\frac{\dot{\varepsilon}_{C}}{\dot{\varepsilon}_{O}}\}^{1/m} = \{\frac{\kappa}{\sigma_{O}(2\pi\kappa)^{1/2}}\}^{1/m}$$
(11)

where Norton's creep law is used and K' is determined by satisfying the equilibrium condition across the plane y=0, the crack plane. For a center cracked plate of width 2W, crack length 2a and thickness B, the equilibrium condition requires that

$$P=B \int_{0}^{W-a} \sigma_{C}^{dx} = B\sigma_{0} \int_{0}^{W-a} \left\{ \frac{K'}{\sigma_{0}(2\pi x)^{1/2}} \right\}^{1/m} dx$$
 (12)

and a simple manipulation gives

$$K' = \sigma_0(2\pi)^{1/2} \left\{ \frac{P}{B\sigma_0} \frac{2m-1}{2m} (W-a)^{-(2m-1)/m} \right\}^m$$

it is of interest to note that for large m, say m \geq 5, (2m-1)/2m = 1 and K' is related to the net section stress as

$$K' = \sigma_0 (2\pi)^{1/2} \left\{ \frac{\sigma_{\text{net}}}{\sigma_0} \right\}^m$$
 (13)

Due to the approximation used, Equation 13 becomes incorrect in units.

Barnby used a critical strain condition at a distance d_c , say the distance of closest creep void or grain boundary fissure, ahead of the main crack as the condition for crack advancement. Since

$$\frac{\dot{\varepsilon}_{\rm C}}{\dot{\varepsilon}_{\rm o}} = \frac{K'}{\sigma_{\rm o}(2\pi x)^{1/2}} \tag{14}$$

by his assumption, he scaled his crack growth rate to creep rate as

$$\frac{da}{dt} = \dot{\varepsilon}_{c} \left(\frac{x}{d_{c}}\right)^{1/2} \tag{15}$$

which is again incorrect in units. Inserting the value of K' he obtained

$$\frac{da}{dt} = \frac{\varepsilon_0}{d_c} \frac{1}{2} \left\{ \frac{[P/B\sigma_0] [(2m-1)/2m]}{(W-a)(2m-1)/2m} \right\}^m$$
(16)

In an attempt to relate his results to the crack growth rate in a linear elastic material, he defined a new parameter K" for which

$$P = \sigma_0 B \int_0^{W-a} \frac{K'' dx}{\sigma_0 (2\pi x)^{1/2}}$$
 (17)

and forced K'' = K' = K' at m = 1. Thus he obtained

$$K/Y(a,W) = K''/N(a,W)$$
 (18)

where N(a,W) is obtained from K'' = K' at m = 1. Finally, the crack growth rate was obtained as

$$\frac{da}{dt} = \left[\dot{\varepsilon}_{0}/\sigma_{0}(2\pi d_{c})^{1/2}\right] K \frac{N(a,W)}{Y(a,W)}, m=1$$
 (19)

His model predicts a faster crack growth in a material which creeps than a linear elastic model (Figure 17).

The approximate analysis used here applies only to the stress component directly ahead of the crack and normal to the crack plane. If damage is not localized the approximation is no good. Since he used the creep exponent m for the crack growth law in an elastic model $da/dt = AK^{m}$ in the comparison shown in Figure 17, the comparison is not meaningful.

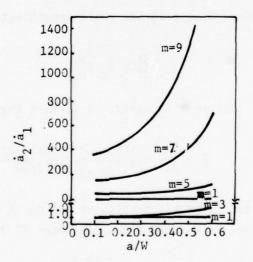


Figure 17. Comparison of the Crack Growth Rate á₂ Obtained by Predictions of the Model, With Crack Growth Rate á₁ Predicted by the Empirical Correlation With the Elastic Stress Intensity Factor (Reference 64)

2. MODEL OF CRITICAL PLASTIC ZONE SIZE (REFERENCE 66)

Assuming that the damage zone ahead of the crack tip is a thin strip, To used the plane stress analysis by Dugdale (Reference 67) where the size of the yield zone is

$$R = C_o \{ Sec \left[\frac{1}{2\pi} \left(\frac{\sigma}{\sigma_v} \right) \right] - 1 \}$$
 (20)

and the yield zone is under uniform biaxial stress of magnitude $\sigma_{\gamma},$ the uniaxial yield stress. The crack tip stresses are therefore characterized by a time-independent analysis for the sake of simplicity.

He then introduced a time-dependent process through the use of Kachanov's (Reference 68) theory of creep rupture, i.e., through the use of the continuity equation

$$d\phi/dt = -C(\sigma/\phi)^{\vee} \tag{21}$$

where C, ν are material constants related to a uniaxial creep test and ϕ is the continuity parameter. For a perfect material ϕ =1, while for a ruptured material ϕ =0. The parameter ν is in general less than the creep exponent and is equal to the inverse slope of a conventional log-log curve of stress against rupture life.

Integrating Equation 21 for constant stress, the rupture time is obtained as

$$t_{R} = T_{K} \left({}^{\sigma}_{K} / {}^{\sigma}_{Y} \right)^{\nu} \phi_{0}^{\nu+1}$$
 (22)

where

 σ_{K} = rupture stress of material in absence of crack

 T_K = rupture time correspond to σ_K

 ϕ_0 = continuity of region before the arrival of a crack with its associated plastic zone.

He used a cumulative damage procedure to determine the initial continuity φ_0 before the arrival of the crack tip plastic zone. Let the half crack-length at time t_n be C_n . At each time interval t_R the crack travels a distance equal to its plastic zone size. Note that the plastic zone size is a function of current crack length and therefore tends to become larger as the crack advances. Using this procedure a first approximation to the crack growth rate was found to be:

$$\frac{dC_n}{dt} = \frac{\ln \sec \frac{\pi}{2} \frac{\sigma}{\sigma_y}}{q_0} \frac{C_0}{t_i} \left(\frac{K}{K_0}\right)^2 \left\{1 + \frac{2q_1 q_0}{\ln \sec \frac{\pi}{2} \frac{\sigma}{\sigma_y}} \ln \frac{K}{K_0}\right\}^2$$
 (23)

where K and K_{Ω} are the current and initial stress intensity factors and

$$q_{0} = \Sigma_{m}(-1)^{m} \left\{ \frac{2(\sec \frac{\pi}{2} \frac{\sigma}{\sigma_{\gamma}} + 1)}{\pi^{2} (0.5 + \sum_{2 \leq m' \leq m} (\sec \frac{\pi}{2} \frac{\sigma}{\sigma_{\gamma}})^{m'-1}} \right\}^{\sqrt{m}/2}$$
(24)

$$q_1 = 0.5(\sigma/\sigma_{\gamma})^{\vee} \tag{25}$$

$$t_i = T_K(\sigma_K/\sigma_Y)^V$$
 (26)

He noted that his derived crack growth rate is of the power law form

$$\frac{dC_n}{dt} = A(K/K_0)^n \tag{27}$$

and made a comparison with data obtained by Siverns and Price (Reference 26). He showed a straight line flot on a log-log scale as given in Figure 18.

If the second term in Equation 23 is taken into consideration, however, a decrease in slope results as K increases or the crack extends. The model does not include work-hardening and hence does not admit a singularity at the crack top. Since both A and n in Equation 27 are

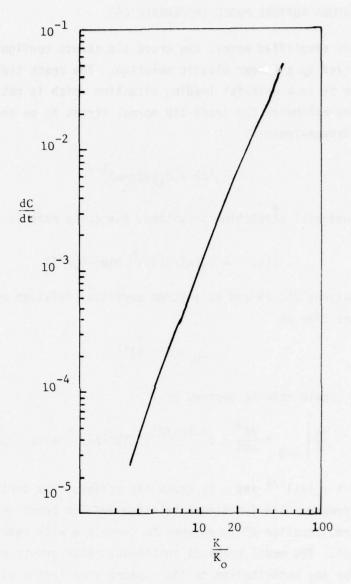


Figure 18. The Crack Growth Rate
Against Stress Intensity
Factor Curve (Reference 66)

stress dependent, the implication here is that either the stress intensity factor K is not a convenient parameter to use in crack growth rate prediction or that the power law is not the proper functional relation for creep crack growth.

3. CONTINUOUS RUPTURE MODEL (REFERENCE 69)

In this simplified model, the crack tip stress configuration is characterized by a linear elastic solution. The crack tip region is assumed to be in a uniaxial loading situation which is not realistic. The authors estimated the crack-tip normal stress to be the same as given by Orowan-Irwin:

$$\sigma_{\mathbf{Y}}(\mathbf{x}) = \sigma_{\mathbf{t}}(\mathbf{a}/\rho + \mathbf{x})^{1/2} \tag{28}$$

Taking a uniaxial stretching situation, the creep rate is calculated as

$$\dot{\varepsilon}(x) = A\{\sigma_{\gamma}(x)/E(T)\}^{m} \exp(-Q_{c}/RT)$$
 (29)

Using Equations 28, 29 and an assumed empirical relation of creep rate and rupture time as

$$t_{R} = B \cdot (\dot{e})^{-X} \tag{30}$$

the crack growth rate is derived as

$$\frac{da}{dt} \bigg|_{X=0} = \frac{2A^{\alpha}}{\alpha mB} \cdot \rho^{1-(\alpha m/2)} \{ K/YE\pi 1/2 \}^{\alpha m} \exp(-\alpha Q_c/RT) \bigg|$$
(31)

where $K = Y \sigma_{\infty}(\pi a)^{1/2}$ and ρ is crack tip radius. The derivation is straightforward. The functional relation between creep rate and temperature, Equation 29 is assumed to correlate with experimental observation. The model predicts continuous crack growth and does not account for any contribution to the rupture time that a point experiences before the arrival of the crack tip. The theory is therefore not complete. Furthermore, Leeuwen (Reference 70) found that the relation (Equation 31) was unreliable for materials other than purely annealed metals.

4. MODEL OF CRITICAL C*-INTEGRAL (REFERENCES 16, 17)

For a material that follows a creep law of the form

$$\dot{\varepsilon}/\varepsilon_0 = A(\sigma/\sigma_0)^n \tag{32}$$

in uniaxial tension, Goldman and Hutchinson (Reference 71) showed that there exists a singularity in strain rate at the crack tip whose amplitude is

$$\dot{K}_{\varepsilon} = \left[C^{*}/A \sigma_{0} \varepsilon_{0} \operatorname{In}\right]^{n/(n+1)}$$
(33)

where I_n is a constant which is tabulated for a given range of n by Hutchinson (Reference 72) and C* is a line integral which was later used by Landes and Begley (Reference 16) to describe creep crack growth and is defined as:

$$C^* = -\frac{1}{da} = \int_{\Gamma} \left[W^* (\dot{\varepsilon}_{mn}) dy - \overline{T} \cdot \frac{\partial \dot{u}}{\partial x} ds \right]$$
 (34)

It can easily be shown that the C* integral is equivalent to:

$$C^* = A \sigma_0 \epsilon_0 K_\sigma \dot{K}_\epsilon I_n$$

$$= A \sigma_0 \epsilon_0 (\dot{K}_\epsilon)^{(n+1)/n} I_n$$
(35)

It should be noted that Equations 33 and 34 apply only to steady state creep and are approximate solutions since the elastic strains are ignored (Reference 65). Nikbin, Webster, and Turner (Reference 17) used the same definition but termed it j.

An experimental determination of C* can be obtained by controlling displacement rate. By monitoring load and crack length at constant displacement rate, a crack growth rate (a) vs. C* plot can be obtained. A data reduction scheme is given in Figure 19 (Reference 16). This experimental procedure requires data from several specimens, say, six to ten.

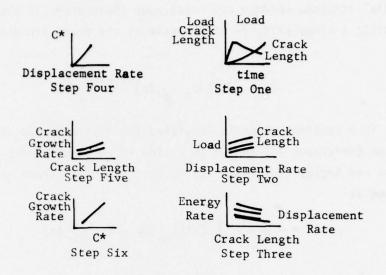


Figure 19. Schematic Showing Six Steps Involved in C* Data Reduction (Reference 16)

Webster (Reference 73) and Nickbin, et al. (Reference 17) employed nonlinear beam theory and derived the following approximate relation for C*:

$$C^* = \frac{1}{B} \cdot \frac{P}{n+1} \cdot \frac{d\Delta}{da}$$
 (36)

in a DCB specimen for conditions of constant loading. The load point displacement is denoted by Δ . Since beam theory was used and no account is taken of deformation ahead of the crack, behind the beam arms, the method can only supply to cases where this deformation is small in relation to the overall deformation seen at the loading points.

In analogy to the theoretical estimation of J. Harper and Ellison (Reference 18) suggested a method for determining C* through limit analysis.

Assuming that the multiaxial creep behavior of the material is given

$$\varepsilon_{i,j} = B_n \phi^n \partial \phi / \partial \phi_{i,j}$$
 (37)

where $\partial \phi(\partial \phi_{ij})$ is an effective stress associated with a flow rule (e.g., Von Mises (σ^*) , Tresca) and is a homogeneous first power function of stress σ_{ij} , and ignoring the singular behavior at the crack tip, they showed that

$$U^* = -\frac{1}{n+1}B_n \left(\frac{P}{mBW}\right)^n P \quad \text{(constant load)}$$
 (38)

$$U^* = \frac{n}{n+1}B_n \left(\frac{P}{mBW}\right)^n P \quad \text{(constant displacement rate)}$$
 (39)

where m is the yield load ratio defined as the tensile limit load of a cracked specimen to the limit load of an uncracked specimen and l is a characteristic length term, for example, the gauge length of a uniaxial specimen which when subjected to a reference stress of (P/mBW) will give a creep dissipation rate identical to that of a cracked specimen.

Differentiating U^* with respect to a and dividing by B, they arrived at

$$C^* = -\frac{n}{n+1} \frac{P\Delta}{BW} \left[\frac{1}{m} \cdot \frac{dm}{d(a/W)} \right]$$
 (40)

Relations of m in term of (a/W) have been derived and given in References 74 and 75. A typical plot for the compact tension specimen is given in Figure 20. The analysis is approximate in nature. In general, it applies to deep notched specimens where fully plastic conditions are achieved. It is therefore not an efficient method to use in studying CCG. For CCG studies determination of C* for limited plasticity is needed.

CRITICAL COD MODEL (REFERENCE 20)

A phenomenological theory of CCG has been developed using the results of a calculation of the time dependent development of the damage zone, termed plastic zone by the author, ahead of crack tip (Reference 20). The whole matrix is assumed to be undergoing creep deformation. The plastic zone is represented by an array of dislocations coplanar

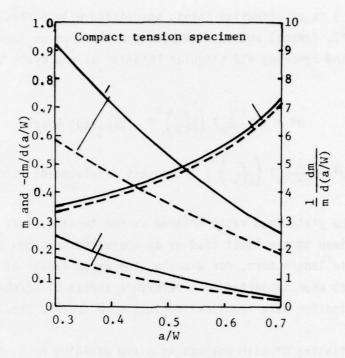


Figure 20. Variation of m, dm/d(a/W), and (1/m) [dm/d(a/W)] With (a/W) (Reference 18)

with the crack as in the model of Bilby, Cotrell and Swinden (Reference 76), henceforth referred to as BCS model.

Vitek showed that at any given point in the plastic zone, the dislocation density rate, $\partial B/\partial t$, is proportional to the negative of $\partial \dot{\epsilon}(x,t)/\partial x$,

$$\frac{\partial B(x,t)}{\partial t} = -h \frac{\alpha \dot{\epsilon}(x,t)}{\partial x}$$
 (41)

where h is the plastic zone width. The plastic zone is confined to a region of length, S, given by

$$S = a \left[sec \left(\frac{\pi \sigma}{2\sigma_1} F - F \right) \right]$$
 (42)

AFML-TR-78-136

where $\sigma_{1}^{\ F}$ is the apparent functional stress. He used an approximate expression for COD as

$$\phi = \frac{4(1-v)}{\pi} a \left(\frac{\sigma_1^F}{G}\right) \ln \sec \frac{\pi}{2} \frac{\sigma}{\sigma_1^F}$$
 (43)

Using a critical crack opening displacement criterion and assuming the crack advances through a fixed distance, be it any distance d, the plastic zone size, or a critical length. Vitek derived the equation of CCG rate as

$$\frac{V}{V_0} = A \cdot K^{\alpha} \tag{44}$$

where V_0 is a constant and A and α depend on σ/G , and regions of the stress relaxation, σ_1^F/σ . Equation 44 may be used for either one of the three modes of loading. A singular integral equation for B is obtained by satisfying the equilibrium conditions. The stress at a point x is

$$\tau(x,t) = \tau^{C+appl\cdot}(x) + \int_{a}^{\infty} \alpha(x',t) \left[\frac{G}{2\pi(1-\nu)} \left(\frac{1}{x-x'} - \frac{1}{x+x'} \right) + \tau^{\dim}(x,x') - \tau^{\dim}(x,-x') \right] dx'$$
(45)

where $\zeta^{\text{C+appl}}$ is the stress due to the crack and the applied stress and ζ^{dim} is the stress due to the image stress field of a unit dislocation which results from zero traction on the crack surface. Vitek (Reference 76) found $\zeta^{\text{C+appl}}$ and ζ^{dim} by conveniently using a conformal mapping function

$$z = \frac{1}{2} a \left(\zeta + 1/\zeta \right) \tag{46}$$

and known results for an elliptical-shaped crack.

It was found that at any time 95% of the dislocations modeling α_{j} depend on σ/G . This equation can be applied to different modes of cracking and different degrees of stress relaxation.

The theory applies when creep is localized at the crack tip. It does not apply when the stress exponent is greater then, say, 5. In general, different exponents α have to be used for three different ranges of K depending upon degree of stress relaxation.

Since the model is basically a uniaxial model, it cannot be readily extended to comparison of experimental data generated using typical two-dimensional specimens such as the compact tension specimen. However, this is the only theoretical development that attempts to describe all three stages of CCG.

6. SUMMARY

There are several models available for a theoretical prediction of the crack growth rate under sustained loading. Depending upon the criteria used for initiation and growth of cracks, these may be any of a number of power law relations between da/dt and various fracture characterization parameters. Although the results are not conclusive as to which parameter should be the best one to use (due to a lack of experimental data) it is of interest to point out the following:

- (1) CCG rate is governed by the material behavior at the given temperature (References 16, 17, 20). Nonlinear analysis is generally required.
- (2) A criterion of initiation of CCG must be determined (References 16, 20, 63, 64, 66).
- (3) For a creep matrix that is governed by Norton's law, there is a creep rate singularly at the crack tip (Reference 71). If the stress exponent, n, is small (say less than five) intensive creep occurs only in the very near vicinity of the crack tip (Reference 20).
- (4) A functional relation between CCG rate and the governing fracture parameter depends on the criterion used to simulate the microscopic mechanism of CCG (References 63, 64, 69).

SECTION V

CONCLUSIONS

- (1) Fatigue crack growth rate is time-independent at high frequency but time-dependent at low frequency. Time-dependent effects on FCG rates come from oxidation and CCG.
- (2) CCG is basically intergranular and closely related to coalescence of microcracks ahead of crack tip with the main crack.
- (3) When the damage ahead of a crack is localized, it is appropriate to use the fracture mechanics approach to analyze the growth of cracks under creep conditions.
- (4) In general, the stress exponent in a uniaxial creep law can be used as an index to differentiate between creep brittle and creep ductile materials. However, this concept requires further verification.
- (5) Lack of data from critically designed experiments on CCG is a basic limitation to the understanding of the phenomenon of CCG.
- (6) Better models are still required to accurately characterize the initiation and growth of cracks at elevated temperatures.

REFERENCES

- J. T. Berling and J. T. Conway, "First International Conference On Pressure Vessel Technology," Delft, Holland, (1969).
- 2. J. T. Berling and Slot, ASTM STP 465 (1968).
- 3. H. D. Solomon and L. F. Coffin, Jr., "Effects of Frequency and Environment on Fatigue Crack Growth in A-286 at 1100°F," ASTM STP 520, (1973), 112.
- 4. H. D. Solomon, "Frequency Dependent Low Cycle Fatigue Crack Propagation Metallurgical Transactions," Vol. 4, (1973), 341.
- P. C. Paris and F. Erdogan, "A Critical Analysis of Crack Propagation Laws," J. Basic Eng. Trans. ASME, Series D, Vol. 85, (1963), 528-534.
- R. M. Gamble and P. C. Paris, "Cyclic Crack Growth Analysis for Notched Structures at Elevated Temperatures," ASTM STP 590, (1976), 345-367.
- 7. R. M. Wallace, C. G. Annis Jr., and D. L. Sims, <u>Application of Fracture Mechanics at Elevated Temperatures</u>, AFML-TR-76-176 Part II, (1977).
- 8. T. Ohmura, R. M. Pelloux, and N. J. Grant, "High Temperature Fatigue Crack Growth in a Cobalt Base Superalloys," Eng. Fracture Mechanics, 5, (1973), 909-922.
- 9. L. A. James, "The Effect of Frequency Upon the Fatigue-Crack Growth of 304 Stainless Steel at 1000°F," ASTM STP 513, (1972), 218-229.
- L. A. James, "Some Questions Regarding the Interaction of Creep and Fatigue," Paper No. 75-WA-MAT-6, J. Eng. Materials and Technology, Trans ASME, (1975).
- 11. M. Gell and G. R. Leverant, "Mechanisms of High Temperature Fatigue," ASTM, STP 520, (1973), 37.
- 12. A. E. Carden, "Parametric Analysis of Fatigue Crack Growth," Proc. Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Paper C230/73, Philadelphia, (1973), Sheffield, U.K. (1974).
- 13. J. Weertman, "Theory of High Temperature Intercrystalline Fracture Under Static or Fatigue Loads, With or Without Irradiation Damage," Metall. Trans., Vol. 5, (1974), 1743-1751.
- S. Floreen, "The Creep Fracture of Wrought Nickel-Base Alloys by a Fracture Mechanics Approach," Metall. Trans. A 6A, (1975), 1741-1749.

- 15. M. Döner, "An Analysis of Elevated Temperature Fatigue and Creep Crack Growth," J. of Engng. for Power, (1976), 473-479.
- J. D. Landes and J. A. Begley," A Fracture Mechanics Approach to Creep Crack Growth ASTM STP 590, (1976), 128-148.
- K. M. Nikbin, G. A. Webster, and C. E. Turner, "Relevance of Nonlinear Fracture Mechanics to Creep Cracking," ASTM STP 601, (1976), 47-62.
- M. P. Harper and E. G. Ellison, "The Use of the C*-Parameter in Predicting Creep Crack Propagation Rates," J. Strain Analysis, 12, No. 3, (1977), 167-179.
- J. R. Haigh, "The Growth of Fatigue Cracks at High Temperature, Under Predominately Elastic Loading," Engng. Fracture Mechanics, (1975), 271-284.
- 20. V. Vitek, "A Theory of the Initiation of Creep Crack Growth," Int. Journ. of Fracture, 13, (1977), 39-50.
- 21. R. Pilkington, D. Hutchinson, and C. L. Jones, "High-Temperature Crack-Opening Displacement Measurements in a Ferrite Steel," Metall. Sci. J. 8, (1974), 237-241.
- R. D. Nicholson and C. L. Formby, "The Validity of Various Fracture Mechanics Methods at Creep Temperatures," Int. Journ. of Fracture, 11, (1975), 595-604.
- 23. R. D. Nicholson, "The Effect of Temperature on Creep Crack Propagation in AISI 316 Stainless Steel," Mater. Sci. Engng. 22, (1976), 1-6.
- C. B. Harrison and G. N. Sandor, High-Temperature Crack Growth in Low-Cycle Fatigue, Eng. Fracture Mech. Vol. 3, (1971), 403-420.
- 25. DeKoning, P. Dobes, and J. Cadek, "On the Kinetics of Stress Induced Growth of Grain Boundary Voids," Scripta Metall. $\underline{4}$, 1005-1008, 1970.
- 26. M. J. Siverns and A. T. Price, "Crack Propagation Under Creep Conditions in a Quenched 2 1/4 Chromium 1 Molybdenum Steel," Int. J. Fracture, Vol. 9, (1973), 199-207.
- 27. C. G. Annis Jr., R. M. Wallace, and D. L. Sims, <u>An Interpolative Model for Elevated Temperature Fatigue Crack Propagation</u>, <u>AFML-TR-76-176</u>, (1976).
- 28. L. A. James, "The Effect of Stress Ratio on the Fatigue-Crack Propagation of Type 304 Stainless Steel," Nuclear Technology, Vol. 16, (1972), 163-170.

- 29. L. A. James, "Hold-Time Effects on the Elevated Temperature Fatigue-Crack Propagation of Type 304 Stainless Steel," Nuclear Technology, Vol. 16, (1972), 521-530.
- 30. J. R. Haigh, "The Mechanisms of Macroscopic High Temperature Crack Growth, I. Experiments on Tempered Cr-Mo-V Steels, II." Review and Re-Analysis of Previous Work, Materials Sci. and Eng. 20, (1975), 213-223, 225-235.
- 31. H. G. Popp and A. Coles, Subcritical Crack Growth Criteria for Inconel 718 at Elevated Temperatures, Proc. Air Force Conf. On Fatigue and Fracture, AFFDL-TR-70-144, (1969), 71.
- 32. F. Garofalo, Trans. AIME, 227, (1963), 351.
- 33. P. G. Shewmon, Diffusion in Solids, McGraw-Hill, New York, (1963).
- R. C. Gifkins, "Grain-Boundary Sliding and Its Accommodation During Creep and Super Plasticity," Metallurgical Transactions, A. Vol. 7A, (1976), 1225-1232.
- 35. P. T. Heald and J. A. Williams," Wedge Crack Growth Enhanced by Vacancy Diffusion Under Creep Conditions," Phil. Mag. Vol. 24, No. 191, (1971), 1215-1220.
- R. N. Steven and R. Dutton, "The Propagation of Griffith Cracks at High Temperatures by Mass Transport Processes," Mater. Sci. Engng, 8, (1971), 220-234.
- 37. R. N. Stevens, R. Dutton, and M. P. Puls, "The Chemical Stress Applied to Creep and Fracture Theories I." A General Approach, Acta Metall. 22, (1974), 629-638
- 38. M. P. Puls, R. Dutton, and R. N. Stevens, Act. Metall., <u>22</u>, (1974), 639.
- 39. D. Hull and D. E. Rimmer, "The Growth of Grain Boundary Voids Under Stress," Philos. Magazine $\underline{4}$, (1959), 673-687.
- 40. M. V. Speight and J. E. Harris, "The Kinetics of Stress-Induced Growth of Grain Boundary Voids," Metall. Sci. J. 1, (1967), 83-85.
- 41. P. Dobes and J. Cadek, "On the Kinetics of Stress Induced Growth of Grain Boundary Voids," Scripta Metall. 4, (1970), 1005-1008.
- 42. P. Dobes, Scripta Met. 7, (1973), 1231.
- 43. R. Raj and M. E. Ashby, "Intergranular Fracture at Elevated Temperature," Acta Metall. 23, (1975), 653-666.

- 44. R. Raj, Met. Trans. 6A, (1975), 1499.
- 45. M. V. Speight and W. Beere, "Vacancy Potential and Void Growth on Grain Boundaries," Metall. Sci., J. 9, (1975), 190-191.
- 46. N. G. Needham and G. W. Greenwood, "The Creep of Copper Under Superimposed Hydrostatic Pressure," Metal. Sci. J. 9, (1975), 258-262.
- 47. N. G. Needham, J. E. Wheatley, and G. W., Greenwood, "The Creep Fracture of Copper and Magnesium," Acta Metall. 23, 23-27, 1975.
- 48. D. A. Kelly, Metal Science, 10, (1976), 57.
- 49. J. W. Hancock, "Creep Cavitation Without a Vacancy Flux," Metal Sci. J. 10, (9), (1976), 297-336.
- H. P. Van Leeuwen, "The Application of Fracture Mechanics to Creep Crack Growth," Engineering Fracture Mechanics, 9, (1977), 951-974.
- 51. C. T. Sims and W. C. Hagel, eds, <u>Superalloys</u> John Wiley, New York, (1972).
- 52. R. W. Hertzberg, <u>Deformation and Fracture Mechanics of Engineering Materials</u>, John Wiley, New York, (1976).
- J. R. Rice and D. M. Tracey, "On the Ductile Enlargement of Voids in Triaxial Stress Fields," J. Mech. Phys. Solids <u>17</u>, (1969), 201-217.
- P. T. Heald, J. A. Williams, and R. P. Harrison, "On the Mechanical Interaction Between a Crack Tip and a Point Defect, "Scripta Metall. <u>5</u>, (1971), 543-546.
- 55. J. R. Rice and Chuang, Acta Metall. 21, (1973), 1625.
- 56. J. G. Kaufman, K. O. Bogardus, D. A. Mauney, and R. C. Malcolm, "Creep Cracking in 2219-T851 Plate at Elevated Temperatures," ASTM STP 590, (1976), 149-168.
- 57. K. Sadananda and P. Shahinian, "Creep Crack Growth in Alloy 718," Metall. Trans. A, Vol. 8A, No. 3, (1977), 439-449.
- 58. S. Taira and R. Ohtani, <u>Creep Crack Propagation and Creep Rupture of Notched Specimens</u>, Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Philadelphia, (1973), Sheffield, (1974).
- 59. R. D. Townsend, The Effect of Structure on the Creep Properties of Low Alloy Ferritic Steels, CEGB Report RD/L/R, (1971), 1740, 1771, CEGB, U.K.

- 60. J. A. Williams and A. T. Price, "A Description of Crack Growth From Defects Under Creep Conditions," Trans. ASME, J. Engg. Materials and Tech., 97, (1975) July, 214-222.
- 61. G. J. Neate and M. J. Siverns, "The Application of Fracture Mechanics to Creep Crack Growth," Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Philadelphia (1973), Sheffield (1974).
- 62. J. L. Kenyon, G. A. Webster, J. C. Radon, and C. E. Turner, Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Paper 156/73, Inst. Mech. Engrs. (1973).
- 63. J. T. Barnby, "Crack Tip Stresses Under Creep Conditions," Eng. Fracture Mech. 6, (1974), 627-630.
- 64. J. T. Barnby, "Crack Propagation During Steady State Creep," Eng. Fracture Mech. 7, (1975), 299-304.
- 65. N. J. Hoff, Q. Application Math., 12, (1954), 49.
- 66, K. C. To, "Phenomenological Theory of Sub-Critical Creep Crack Growth Under Constant Loading in an Inert Environment," Int. Journ. Fracture, 11, (1975), 641-648.
- 67. D. S. Dugdale, "Yielding of Steel Sheets Containing Slits," J. Mech. Phys. Solids, 8, (1960), 100-108.
- 68. L. M. Kachanov, "On Rupture Time Under Conditions of Creep," Izv. Akad., USSR, Otd. Teckhn. Nauk, No. 8, (1958), 26-31.
- 69. S. Purnshothaman and J. K. Tien, "A Theory of Creep Crack Growth," Scripta Metall., 10, (1976), 663-666.
- 70. H. P. Van Leeuwen, Relating Rupture Time to Minimum Creep Rate and Ductility, NLR Report M, 2112, Feb. (1966).
- 71. N. L. Goldman, and J. W. Hutchinson, <u>Fully Plastic Crack Problems: The Center-Cracked Strip Under Plane Strain</u>, Rep. #DEAP A-7, Harvard Univ., (1974).
- 72. J. W. Hutchinson, "Singular Behavior at the End of a Tensile Crack in a Hardening Material," J. Mech. Phys. Solids, 16, (1968), 13-31.
- 73. G. N. Webster, "The Application of Fracture Mechanics to Creep Cracking," Conf. on Mechanics and Physics of Fracture, Institute of Physics, England, (1975).

- 74. D. J. F. Ewing and C. E. Richards, "The Yield Point Loads of Singly-Notched Pin-Loaded Tensile Strips," J. Mech. Phys. Solids, 22, (1), (1974), 27-36.
- 75. J. R. Haigh and C. E. Richards, <u>Yield Point Loads and Compliance Functions of Fracture Mechanics Specimens</u>, C.E.G.B. C.E.R.L. Report RD/L/M461, (1974).
- 76. B. A. Bilby, A. H. Cotrell and K. H. Swinden, Proc. Roy. Soc. A272, (1963), 304.
- 77. V. Vitek, "Computer Simulation of the Development of Plastic Zone Ahead of a Crack Under Creep Conditions," Nucl. Metall. Vol. 20, Pt. 1-2 (1976), Proc. Int. Conf. on Computer Simulation for Mater. Application, Maryland, (1976), Pt. 2, (1976), 909-916.
- 78. K. Robson, "Creep Crack Growth in Two Carbon Steels at 450°C," Verein Deutscher Eisenhuettentente, Int. Conf. on Creep Resistance in Steel, Dussel dorf, (1972).
- 79. D. V. Thornton, G. E. C. (Whetston) Rept. CML, 18 (1972).
- 80. J. D. Landes and R. P. Wei, "Kinetics of Subcritical Crack Growth and Deformation in a High Strength Steel," Trans. ASME, J. Engng. Mater. Tech. 95, (1973), 2-9.
- 81. E. G. Ellison and D. Walton, "Fatigue Creep and Cyclic Creep Crack Propagation in a lCr-Mo-V Steel," Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Philadelphia/Sheffield, (1973)/(1974).
- 82. R. Donat and L. S. Fu, "Stable Crack Growth in IN-100 at Elevated Temperature," Eleventh National Conference on Fracture Mechanics, Blacksburg, VA, June 1978.

BIBLIOGRAPHY

Adamson, J. M. and Martin, J. W., "Relation Between Crack Austenitic Stainless Steel," Metallography Vo. 9, No. 6, pp. 525-539, 1976.

Atanmo, P. N., "Creep-Fatigue Interaction During Crack Growth," Fatigue at Elevated Temperatures, pp. 157-165, 1973.

Coleman, M. C., Fidler, R., and Williams, J. A., "Crack Growth Monitoring in Pressure Vessels Under Creep Conditions," Weld Inst., Abington, Cambridge, England 40-44, 1976.

Dugdale, D. S., "Yielding of Steel Sheets Containing Slits," J. Mech. Phys. Solids, $\underline{8}$, 1960.

Floreen, S. and Kane, R. H., "A Critical Strain Model for the Creep Fracture of Nickel-Base Superalloys," 7a, 1157-1160, 1976.

Goodall, I. W. and Chubb, E. J., "The Creep Ductile Response of Cracked Structures," Int. Journ. of Fracture, 12, 289-303, 1976.

Heald, P. T., "General Approach to Creep Failure Resulting from Wedge Crack Growth," Phil. Mag. 20, (165), 635-639, 1969.

Heald, P. T. and Williams, J. A., "Creep Failure Resulting from Wedge Crack Growth," Phil. Mag. 22, (179), 1095-1100, 1970.

Jones, G. T., "Subcritical Creep-Crack Growth Problems in Turbo-Generator Material," British Steel Corp. Conf. on Mechanics and Mechanisms of Crack Growth, London, 323-326, 1976.

Johnson, H. H., "Materials Standards," Vol. 5, 442-445, 1965.

Kashiwaya, H., Ishimatsu, M., Yanuki, T., Arima, N., and Arii, M., "Creep Crack Propagation in Aluminum-Cooper-Magnesium Alloy," Proc. Symp. Mech. Behavior of Materials, Kyoto, Japan, August 1974, Vol. 1, 467-477, (1974) Soc. of Mater. Sci., Kyoto, Japan, 1974.

Koterazawa, R., "Applicability of the Linear Fracture Mechanics to Creep-Crack Propagation," Int. J. Fracture, 11, 1060-1062, 1975.

Lange, F. F., "Interrelations Between Creep and Slow Crack Growth for Tensile Loading Conditions," Int. J. Fracture, 12, 739-744, 1976.

Larson, J. M. and Floreen, S., "Metallurgical Factors Affecting the Crack Growth Resistance of a Superalloy," Metall. Trans. A, Vol. 8A, No. 1, 51-55, 1977.

McClintock, F. A., "Continuum Description of Cracks at Macro and Micro Levels and Interaction of Crack Morphology with Environment," Report, Massachusetts Institute of Technology, 1973.

BIBLIOGRAPHY (CONTINUED)

Ohji, K., Ogura, K., and Kubo, S., "Creep Stress Analysis of a Notched Body Under Longitudinal Shear and Its Application to Crack Propagation Problems," Proc. Symp. Mech. Behavior of Materials, Kyoto, Japan, August 21-24, 1974, Vol. 1, 455-466 (1974) Soc. of Material Sci., Kyoto, Japan, 1974.

Ohtani, R. and Nakamura, S., "Crack Propagation in Creep-Finite Element Analysis," J. Soc. Mater. Sci., Japan, Vol. 24, No. 275, 738-745, 1976.

Sasaki, R., Shiga, M., Hataya, F., and Shintaro, T., J. Soc. Mater. Sci., Japan, 25, (270), 236-240, 1976.

Shahinian, P. and Sadananda, K., "Crack Growth Behavior Under Creep-Fatigue Conditions in Alloy 718," Creep-Fatigue Interaction, ASME, New York, 365-390, 1976.

Siverns, M. J. and Price, A. T., Nature, Vol. 228, 5273, 760-761, 1970.

Turner, C. E. and Webster, G. A., "Application of Fracture Mechanics to Creep Crack Growth," Int. J. Fracture 10, 455-459, 1974.

Twigg, R. J. and Taplin, D. M. R., "Creep Fracture of HK 40 Petrochemical Furnace Tube Steel," Interam Conf. on Mater. Technology 4th Proc., Caracas, Venez., 107-118, 1975.

Williams, D. N., "Subcritical Crack Growth Under Sustained Load," Metall. Trans. $\underline{5}$, 11, 2351-2358, 1974.

Williams, D. N., "Effect of Specimen Thickness on Subcritical Crack Growth Under Sustained Load," Mater. Sci. Eng. Vol. 18, No. 1, 149-155, 1975.

Williams, D. N., "Review of Subcritical Crack Growth Under Sustained Load," AIIA J. Vol. 14, No. 3, 342-347, 1976.

Williams, J. A., "Description of Crack Growth from Defects Under Creep Conditions," Trans. ASME, H. J. Engng. Mater. Technology 97, 214-222, 1975.

Wilson, R. N., "Creep Fracture Mechanisms in Aluminum Alloys," Inst. of Metall. Sci. 2, No. 10, 103-111, 1973.